increases, sublimation, chemical reaction, dissociation, and ionization occur. In order to find the equilibrium composition, the method used is that of minimizing the free energy. The thermodynamic functions of the equilibrium mixture based on 1 mol of initial reactant are calculated by the equations for an ideal mixture.

The enthalpy composition charts are then constructed with the calculated values of $H^\circ(P, T, \psi)$. Taking H° as the ordinate and ψ as the abscissa, the constant temperature and entropy lines are shown in the charts. The carbon saturation line is also indicated, having been determined for given ψ by the temperature at which C_β disappears in the equilibrium composition. For the pressures 0·001, 0·01, 0·10, 1·0, 10, and 100 atmospheres, enthalpy-composition charts are given in Figs. 1 through 6, respectively.

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THREE-FLUID HEAT EXCHANGER EFFECTIVENESS

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NOMENCLATURE

A_1, A_2, A_3,	areas as illustrated in Fig. 1;
$A_{(1)}$,	variable area corresponding to surface 1;
A_k,	longitudinal heat conduction area;
$B_1, \ldots, B_6$,	functions defined by equations (11–14), (20) and (21);
c_p,	specific heat at constant pressure;
C,	$= \dot{m}c_p$, capacity rate;
e_c,	$= (T_{c_2} - T_{c_1})/(T_{h_1} - T_{c_1})$, cold fluid temperature ratio;
e_i,	$= (T_{i_2} - T_{i_1})/(T_{h_1} - T_{i_1})$, intermediate fluid temperature ratio;
ϵ,	heat exchanger effectiveness;
k,	thermal conductivity of the separating surface material;
L,	length of the separating surface in the longitudinal direction;
$\dot{m}$,	mass flow rate;
N_{tu},	$= U_1A_1/C_c$, dimensionless number of heat transfer units;
$\dot{Q}$,	heat-transfer rate;
r_2, r_3,	roots of the characteristic equation;
R_1, R_2, R_3,	$= 1/U_1A_1,\ 1/U_2A_2,\ 1/U_3A_3$, thermal resistances;
S_2, S_3,	functions defined by equations (15) and (16);
T_h, T_i, T_c,	fluid temperatures;
$T_{h_1}, T_{i_1}, T_{c_1}$,	inlet fluid temperatures;
$T_{h_2}, T_{i_2}, T_{c_2}$,	outlet fluid temperatures;
T_{ho},	hot fluid temperature defined by equation (4);
U_1, U_2, U_3,	over-all heat-transfer coefficients (see Fig. 1);
X,	$= (T_{h_1} - T_{i_1})/(T_{h_1} - T_{c_1})$, inlet temperature ratio.

Subscripts

c,	cold fluid (fluid with the lowest inlet temperature);

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h, hot fluid (fluid with the highest inlet temperature);
i, intermediate fluid.

COMMON heat exchanger designs involve transfer of energy as heat between pairs of fluid streams; however, heat exchange between three fluids is required in such applications as air separation systems, ammonia gas synthesis systems, etc. The purpose of this communication is to present analytical relationships between the design variables (the NTU-effectiveness relationships) for a general three-fluid heat exchanger, in which all three streams are in thermal communication.

The NTU-effectiveness design method [1], commonly used in two-fluid heat exchanger design, was extended by Sorlie [2] to the restricted case of three-fluid heat exchangers of the concentric-tube and plate-fin types, in which the intermediate and cold streams were thermally isolated from each other. The present investigation was an extension of Sorlie's work by treating the case in which all three streams were in thermal communication, as in many three-fluid heat exchangers used in cryogenic systems.

A schematic of the three-fluid heat exchanger considered in this investigation is shown in Fig. 1. The cold and intermediate streams flow in parallel, while the hot stream flows in the opposite direction. The analysis is also valid for the situation where the cold stream flows counter to the hot and intermediate streams. The following mathematical model (set of assumptions or idealizations) was formulated

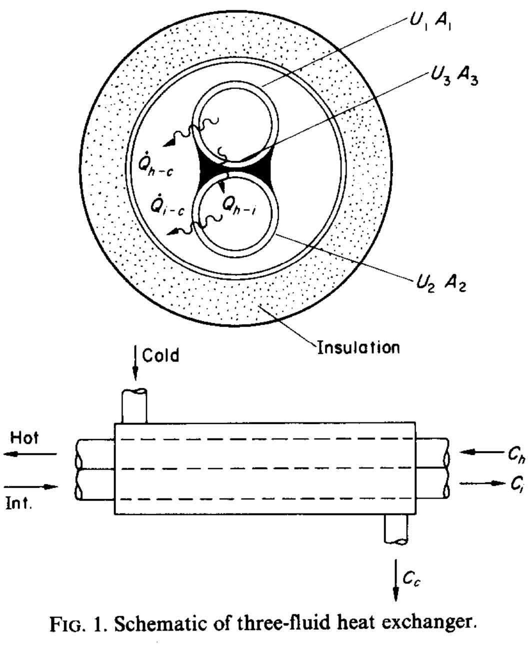

FIG. 1. Schematic of three-fluid heat exchanger.

for the physical problem: (a) There is no heat exchange between the exchanger and the surroundings. (b) Steady state prevails. (c) The physical properties of the fluids are essentially constant. (d) There is no longitudinal conduction along the separating surfaces. (e) Perfect mixing occurs in each flow passage.

In writing the expressions for the heat exchanger effectiveness, two cases must be treated: (a) C_c and C_i less than C_h, and (b) C_h less than C_i and C_c. In the first case, the maximum heat transfer occurs for an infinite-area heat exchanger in which the outlet temperatures of the intermediate and cold streams are equal to the inlet temperature of the hot stream.

$$\dot{Q}_{max} = C_i(T_{h_1} - T_{i_1}) + C_c(T_{h1} - T_{c_1}). \quad (1)$$

The actual heat-transfer rate is

$$\dot{Q}_{actual} = C_i(T_{i_2} - T_{i_1}) + C_c(T_{c_2} - T_{c_1}). \quad (2)$$

Using these two expressions in the definition for heat exchanger effectiveness and expressing the result in dimensionless form, the following expression is obtained.

$$\epsilon = \frac{\dot{Q}_{actual}}{\dot{Q}_{max}} = \frac{(C_i/C_c)e_i X + e_c}{(C_i/C_c)X + 1}. \quad (3)$$

For the second case, in which the capacity rate of the hot stream is smallest, the hot fluid will leave an infinite-area heat exchanger at a temperature T_{ho}, given by

$$T_{ho} = \frac{(R_1/R_3)T_{i_1} + T_{c_1}}{(R_1/R_3) + 1}. \quad (4)$$

The maximum possible heat exchange for the second case is

$$\dot{Q}_{max} = C_h(T_{h_1} - T_{ho})$$

so that the expression for the heat exchanger effectiveness becomes

$$\epsilon = \frac{[(C_c/C_h)e_c + (C_i/C_h)e_i X][(R_1/R_3) + 1]}{(R_1/R_3)X + 1}. \quad (5)$$

In order to complete the solution of the problem, relationships between the temperature ratios e_c and e_i and the other design variables (N_{tu}, X, the capacity rates, and the thermal resistances) were developed. By applying the First Law of Thermodynamics to a differential element in the heat exchanger, the following set of differential equations for the fluid temperatures was obtained.

$$C_c \frac{dT_c}{dA_{(1)}} + C_i \frac{dT_i}{dA_{(1)}} - C_h \frac{dT_h}{dA_{(1)}} = 0 \quad (6)$$

$$C_c \frac{dT_c}{dA_{(1)}} - U_1(T_h - T_c) - U_2(A_2/A_1)(T_i - T_c) = 0 \quad (7)$$

$$C_i \frac{dT_i}{dA_{(1)}} - U_3(A_3/A_1)(T_h - T_i) + U_2(A_2/A_1)(T_i - T_c) = 0. \quad (8)$$

The details of the solution of these equations are given in

[3]. The final results for the temperature ratios e_c and e_i are as follows.

$$e_i = \frac{B_3}{X}\left[\frac{(B_4 - 1) - (1 - X)(B_2 - 1)}{(B_1 - 1)(B_4 - 1) - (B_2 - 1)(B_3 - 1)}\right] \times (\exp[r_2A_1] - 1) + \frac{B_4}{B_4 - 1} \times \left\{1 - X - \frac{(B_3 - 1)[(B_4 - 1) - (1 - X)(B_2 - 1)]}{(B_1 - 1)(B_4 - 1) - (B_2 - 1)(B_3 - 1)}\right\} \times (\exp[r_3A_1] - 1) \quad (9)$$

$$e_c = \left[\frac{(B_4 - 1) - (1 - X)(B_2 - 1)}{(B_1 - 1)(B_4 - 1) - (B_2 - 1)(B_3 - 1)}\right] \times (\exp[r_2A_1] - 1) + \left\{\frac{1 - X}{B_4 - 1} - \left(\frac{B_3 - 1}{B_4 - 1}\right) \times \left[\frac{(B_4 - 1) - (1 - X)(B_2 - 1)}{(B_1 - 1)(B_4 - 1) - (B_2 - 1)(B_3 - 1)}\right]\right\} \times (\exp[r_3A_1] - 1) \quad (10)$$

where:

$$B_1 = \left[\frac{S_2}{1 + (R_1/R_2)(C_h/C_i)}\right]\exp[r_2A_1] \quad (11)$$

$$B_2 = \left[\frac{S_3}{1 + (R_1/R_2)(C_h/C_i)}\right]\exp[r_3A_1] \quad (12)$$

$$B_3 = \frac{(C_h/C_i)S_2}{1 + (R_1/R_2)(C_h/C_i)} - \frac{C_c}{C_i} \quad (13)$$

$$B_4 = \frac{(C_h/C_i)S_3}{1 + (R_1/R_2)(C_h/C_i)} - \frac{C_c}{C_i} \quad (14)$$

$$S_2 = \frac{r_2A_1}{N_{tu}} + \frac{R_1}{R_2}\left(1 + \frac{C_c}{C_i}\right) + 1 \quad (15)$$

$$S_3 = \frac{r_3A_1}{N_{tu}} + \frac{R_1}{R_2}\left(1 + \frac{C_c}{C_i}\right) + 1. \quad (16)$$

The quantities r_2 and r_3 are the roots of the characteristic equation for the system of differential equations (6), (7), and (8), given by

$$\left.\begin{matrix} r_2A_1 \\ r_3A_1 \end{matrix}\right\} = -\tfrac{1}{2}N_{tu}\left[\left(1 - \frac{C_c}{C_h}\right) + \frac{R_1}{R_2}\left(1 + \frac{C_c}{C_i}\right) + \frac{R_1}{R_3}\left(\frac{C_c}{C_i} - \frac{C_c}{C_h}\right)\right] \pm \tfrac{1}{2}N_{tu}\left\{\left[\left(1 - \frac{C_c}{C_h}\right) + \frac{R_1}{R_2}\left(1 + \frac{C_c}{C_i}\right) + \frac{R_1}{R_3}\left(\frac{C_c}{C_i} - \frac{C_c}{C_h}\right)\right]^2 - 4\frac{C_c}{C_i}\frac{R_1}{R_2}\left(1 - \frac{C_i}{C_h} - \frac{C_c}{C_h}\right) \times \left(1 + \frac{R_1}{R_3} + \frac{R_2}{R_3}\right)\right\}^{\frac{1}{2}}. \quad (17)$$

Although the expressions for the temperature ratios e_c and e_i are rather involved, the complexity of the expressions is not as serious a disadvantage as it would seem at first glance, since the expressions are easily handled by a digital computer. A typical computer program is given in [3]. Some representative curves for the temperature ratios are presented in Figs. 2–4. More complete tabulated data are given in [3] also.

For the special case when $C_c/C_h + C_i/C_h - 1 = 0$, the expressions for the temperature ratios become indeterminate. In this case the solution is found to be the following.

$$e_i = \frac{(X - B_4) + B_2(1 - X) + B_4[B_6 - B_5(1 - X)](\exp[r_3A_1] - 1)}{X[B_6(B_2 - 1) - B_5(B_4 - 1)]} \quad (18)$$

$$e_c = \frac{(X - B_4) + B_2(1 - X) + [B_6 - B_5(1 - X)](\exp[r_3A_1] - 1)}{B_6(B_2 - 1) - B_5(B_4 - 1)} \quad (19)$$

where:

$$B_5 = \frac{1}{N_{tu}} - (R_1/R_2)B_6 + 1 \quad (20)$$

$$B_6 = \frac{1 - (R_3/R_1)(C_i/C_c)}{N_{tu}(1 + R_1/R_2 + R_3/R_2)} \quad (21)$$

$$r_3A_1 = -N_{tu}\left[\left(1 - \frac{C_c}{C_h}\right) + \frac{R_1}{R_2}\left(1 + \frac{C_c}{C_i}\right) + \frac{R_1}{R_3}\left(\frac{C_c}{C_i} - \frac{C_c}{C_h}\right)\right]. \quad (22)$$

A typical curve for this special case is given in Fig. 5.

In order to verify the mathematical solution for the heat exchanger performance, a simple dual-tube three-fluid heat exchanger was constructed. The intermediate and hot fluids flowed through two $\frac{7}{8}$-in O.D. copper tubes brazed together longitudinally. The dual-tube arrangement was enclosed within a 2·50-in I.D. by 66-in long copper tube, and the cold fluid flowed in the annular space between the smaller and larger tubes. The entire unit was insulated to minimize heat exchange with the surroundings. For convenience, water was used as the test fluid. Inlet and outlet temperatures were measured by copper–constantan thermocouples inserted into the fluid stream in mixing chambers,

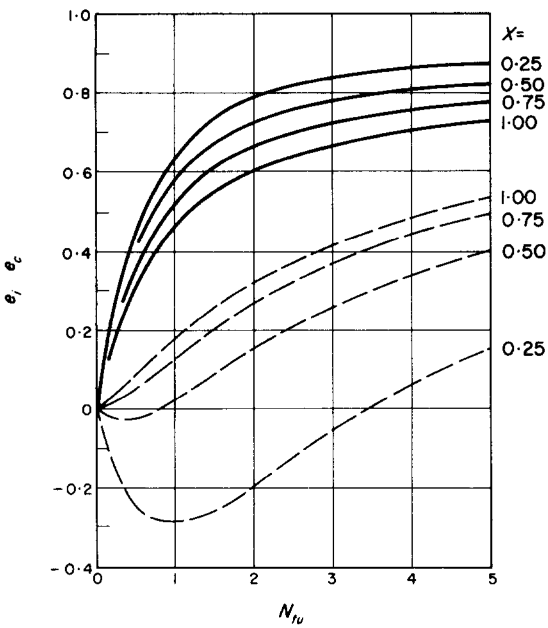

FIG. 2. Temperature ratios. $R_1/R_2 = 0{\cdot}50$; $R_1/R_3 = 0{\cdot}25$; $C_c/C_h = 0{\cdot}50$; $C_i/C_h = 0{\cdot}75$; ——— cold stream, e_c; — — — e_i.

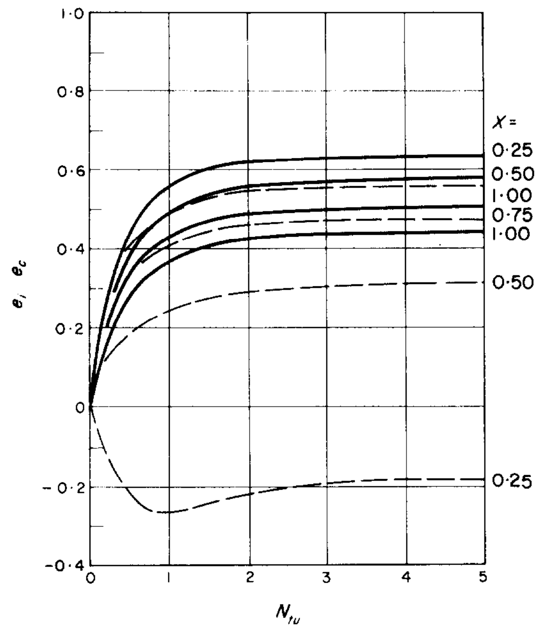

FIG. 4. Temperature ratios. $R_1/R_2 = 0{\cdot}75$; $R_1/R_3 = 2{\cdot}00$; $C_c/C_h = 1{\cdot}00$; $C_i/C_h = 1{\cdot}00$; ——— e_c; — — — e_i.

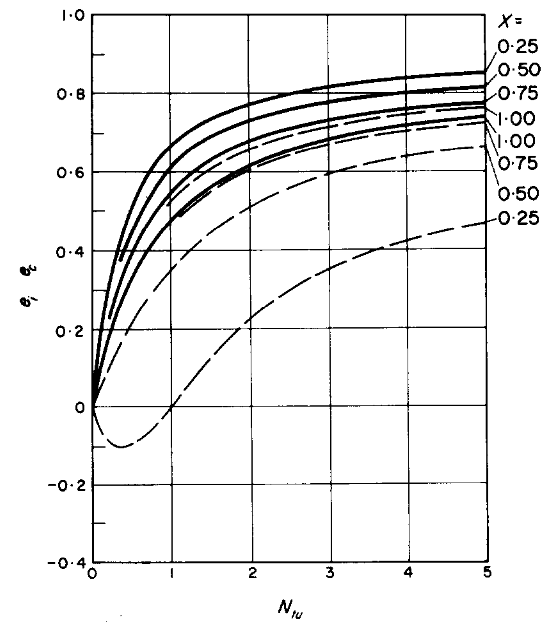

FIG. 3. Temperature ratios. $R_1/R_2 = 1{\cdot}00$: $R_1/R_3 = 2{\cdot}00$; $C_c/C_h = 0{\cdot}50$; $C_i/C_h = 0{\cdot}75$; ——— e_c; — — — e_i.

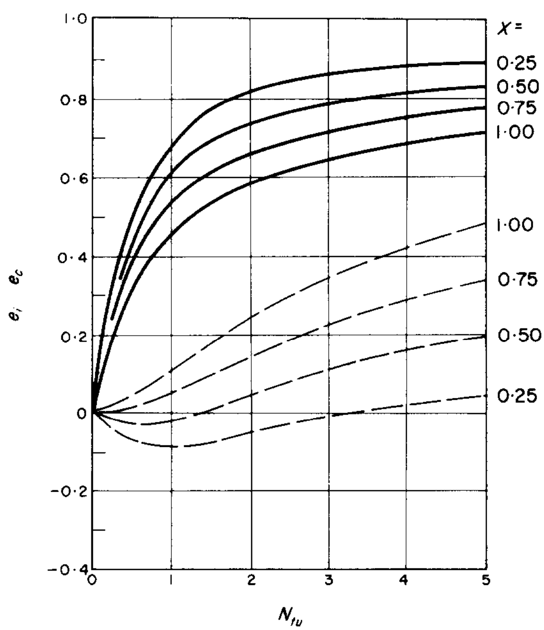

FIG. 5. Temperature ratios for the special case ($C_c/C_h + C_i/C_h = 1$). $R_1/R_2 = 0{\cdot}75$; $R_1/R_3 = 1{\cdot}50$; $C_c/C_h = 0{\cdot}25$; $C_i/C_h = 0{\cdot}75$; ——— e_c; — — — e_i.

so that the mixed-mean temperature was determined. Hot water at approximately 140°F was mixed with cold water at approximately 70°F to produce the intermediate-temperature stream. The inlet temperature of the intermediate stream was varied by varying the ratio of hot and cold water mixed. Mass flow rates were measured by collecting the liquid in weighing vessels over a measured length of time. The thermal resistances between the streams was obtained by making auxiliary runs at the same mass flow rates with only two fluids flowing. The hot and cold streams were paired, then the cold and intermediate, and finally the hot and intermediate streams were paired to obtain R_1, R_2, and R_3 respectively. This procedure is described in detail in [3].

Since all fluid temperatures were measured, the temperature ratios and heat exchanger effectiveness could be calculated directly from the experimental measurements. The final experimental results are presented in Table 1. Detailed experimental data is recorded in [3].

It was noted from both the analytical and experimental results that negative values were obtained for the temperature ratio e_i for small values of the inlet temperature ratio X. From its definition, a negative value of e_i implies that the intermediate stream leaves the exchanger at a lower temperature than that at which it entered, or the cold stream is, in effect, cooling both the hot stream and the intermediate stream. Since the intermediate stream temperature never falls below the cold stream temperature, no violation of the Second Law of Thermodynamics is involved, and the occurence of negative e_i is a physical reality. The fact that negative e_i occurs for small inlet temperature ratios X is to be expected, since a small value of X implies that the intermediate stream enters at a temperature near that of the hot stream at inlet; therefore, one would expect that the intermediate stream would be more likely to be cooled than heated under this condition.

Table 1. Experimental results

X	N_{tu}	Thermal resistances		Capacity rate ratios		e_i		e_c		ϵ	
		R_1/R_2	R_1/R_3	$C_c(C_h$	C_i/C_h	expt.	calc.	expt.	calc.	expt.	calc.
0·756	0·197	3·68	1·066	0·383	0·682	0·055	0·044	0·205	0·243	0·118	0·128
0·281	0·176	3·08	0·743	0·938	1·579	−0·194	−0·424	0·177	0·363	0·058	0·111
0·068	0·121	4·14	1·106	1·723	1·890	−1·210	−3·593	0·197	0·370	0·339	0·324
0·486	0·151	1·16	0·757	0·953	0·760	−0·068	−0·054	0·169	0·192	0·054	0·084
0·471	0·136	0·487	0·707	1·238	1·061	0·051	0·014	0·146	0·143	0·264	0·236

Note: The Reynolds number range for the experimental data is as follows:
Cold stream (in annular passage) 900–2200; Intermediate stream (in $\frac{7}{8}$-in tube) 15000–28000. Hot stream (in $\frac{7}{8}$-in tube) 11000–39000. Equivalent diameter for cold stream in D_e = 4(cross sectional area)/(heated perimeter).

There are two points in the mathematical model upon which the analytical solution is based that require examination when applying the solutions in design: (a) the assumption of constant fluid properties, and (b) the assumption of negligible longitudinal conduction. When wide temperature ranges are involved, the assumption of constant fluid properties is weak, although this assumption is also used in two-fluid heat exchanger design [4]. For the case in which fluid property variations cannot be ignored (such as when the state of the fluid passes through the vicinity of the critical point), the designer must solve the specific problem numerically (no general solution is possible), or he may use the results presented in this work by analyzing the heat exchanger in sections in which the fluid properties do not vary widely. An approximate method for analysis of a three-fluid heat exchanger when variation of fluid properties is significant is outlined by Lenfestey [5].

The assumption of negligible longitudinal heat conduction along the separating surfaces of the streams is generally valid except when the heat exchanger effectiveness is high (above about 0·90) or when the length of the flow passage is short (actually when the parameter kA_k/C_cL is large). This aspect of the problem is one which deserves further investigation, since heat exchanger effectivenesses of 0·95 or higher are not uncommon in cryogenic systems.

As shown in Table 1, the agreement between analytical and experimental values for the heat exchanger effectiveness is satisfactory. Since a rather small laboratory model was used in the experiments, the N_{tu} values were small (N_{tu} gives a measure of the "size" of the exchanger). Note that negative values of e_i were measured experimentally, in agreement with the analytical predictions.

It was felt that the experimental determination of the thermal resistances was responsible for the major differences between the experimental and analytical results, since experimental temperature and flow rate measurements were accurate within ±2 per cent. When all three fluids were flowing in the primary runs, the circumferential conduction through the tubes could have been somewhat different from the case when only two fluids were flowing in the secondary runs used to measure the thermal resistances. A difference in circumferential conduction would result in a difference in the thermal resistances. This is a second aspect of the heat exchanger design problem which deserves further investigation.

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